

# Lecture 7

Effects at Charged Interfaces

# In this lecture...

- Experimental aspects of charged interfaces: characterization of the double layer and the applications

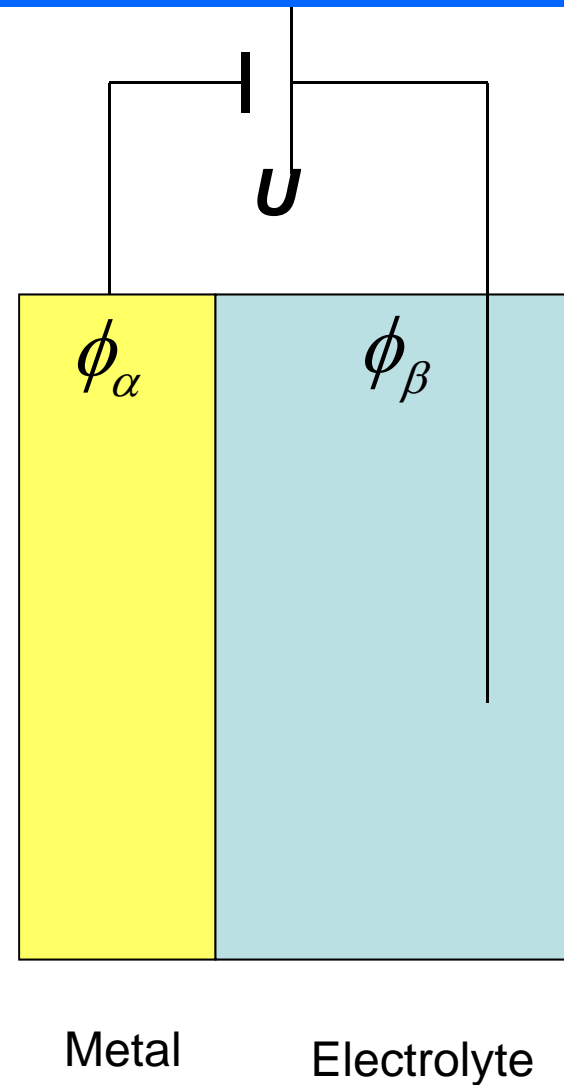
# Electrocapillarity

- Gibbs-Duhem equation for the case of electrified interface:

$$d\gamma = -\sum_i \Gamma_i d\mu_i^* - \Gamma_e d\mu_e^*$$

adsorbed ions  
in the  
electrolyte

electrons in the  
metal attracted  
to the interface



# Electrocapillarity

$$d\gamma = -\sum_i \Gamma_i d\mu_i^* - \Gamma_e d\mu_e^*$$

- electrochemical potential:

$$\mu_i^* = \mu_i + Z_i F d\phi^\beta \quad \text{ions of charge } Z_i.$$

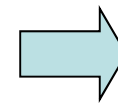
$$\mu_e^* = \mu_e - F d\phi^\alpha \quad \text{electrons}$$

- electroneutrality:

$$\sum_i \Gamma_i z_i = \Gamma_e$$

Lippman equation

$$d\gamma = -\sum_i \Gamma_i d\mu_i - \Gamma_e d\mu_e - \sigma \cdot d(\phi^\beta - \phi^\alpha)$$



$$\frac{\partial \gamma}{\partial U} = -\sigma$$

# Electrocapillarity

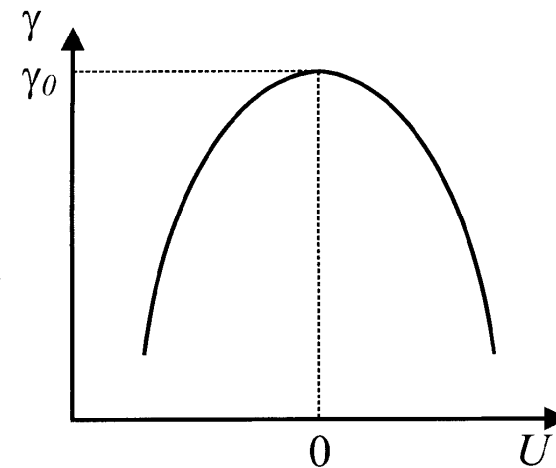
- Lippman equation

$$\frac{\partial \gamma}{\partial U} = -\sigma$$

$$-\frac{\partial^2 \gamma}{\partial U^2} = \frac{\partial \sigma}{\partial U} = C$$

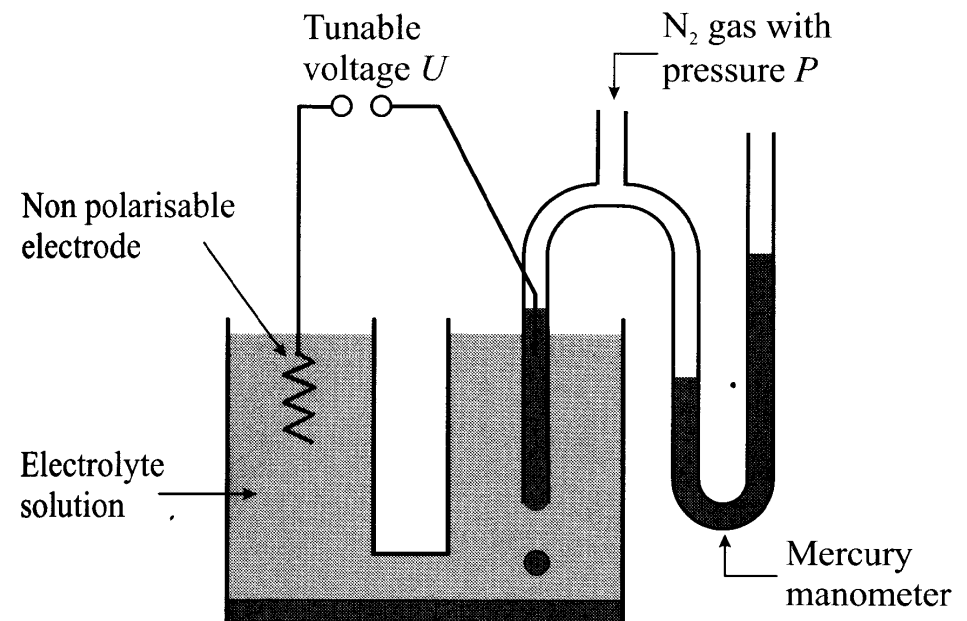
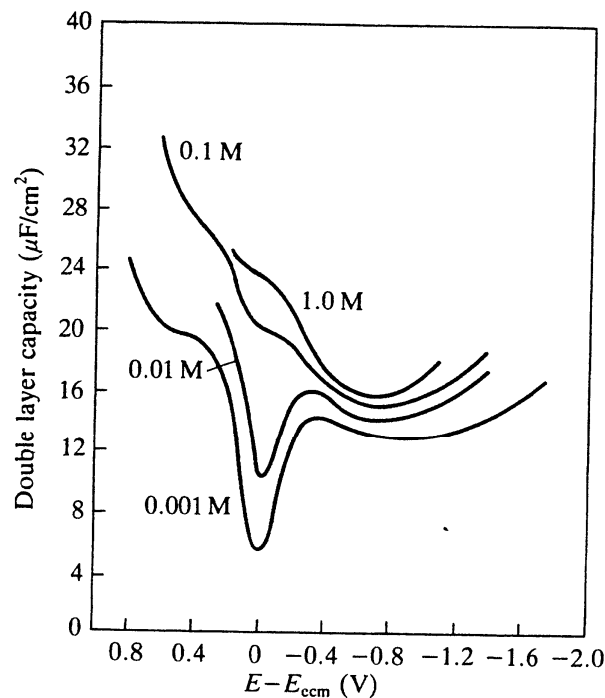
- Assuming capacity constant:

$$\gamma = \gamma_0 - \frac{1}{2}CU^2$$



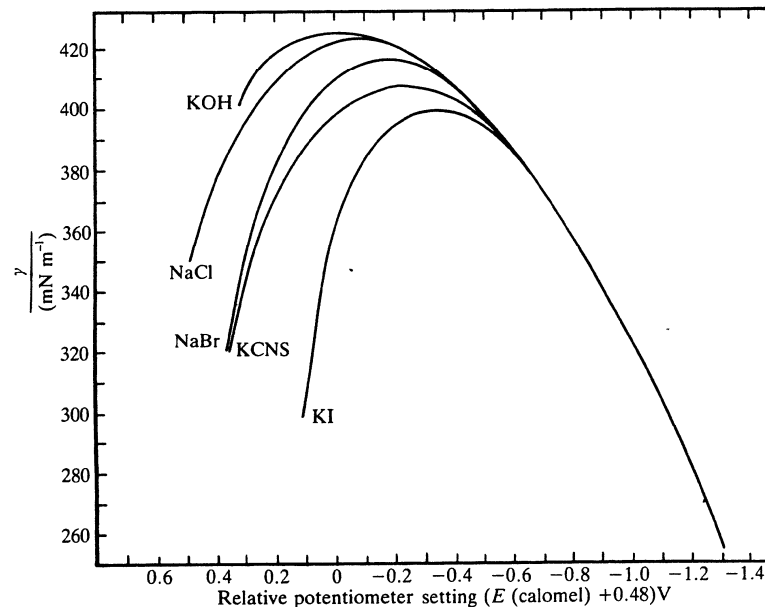
# Electrocapillarity

- Measurements on Dropping mercury electrode (DME)
  - surface tension can be determined using maximal bubble pressure method
  - best arrangement in terms of interpretation



# Examples of charged surfaces: Mercury

- Surface tension of mercury can be easily measured
- Typically a parabola is observed,
  - width of the parabola decreases with salt concentration,
  - minimum position stays the same
- In the solutions of KI, KOH, KCNS a shift of the minimum is observed due to preferential binding of anions for mercury



# Examples of charged surfaces: AgI

- as AgI is slightly soluble:



- due to preferential adsorption of  $I^-$  ions to the surface it acquires negative charge. Small change in the concentration of one of the “potential determining ions” is sufficient to change the point of zero charge.

- In equilibrium:

$$\mu_L^0(Ag^+) + RT \ln a_L(Ag^+) + F\phi_L = \mu_C^0(Ag^+) + RT \ln a_C(Ag^+) + F\phi_C$$

- at the point of zero charge:

$$\mu_L^0(Ag^+) + RT \ln a_L^{pzc}(Ag^+) = \mu_C^0(Ag^+) + RT \ln a_C^{pzc}(Ag^+) + F\Delta\chi^{pzc}$$

- subtracting:

$$\psi = \frac{RT}{F} \ln \frac{a_L(Ag^+)}{a_C^{pzc}(Ag^+)}$$

increase of the concentration  
by x10 changes surface  
potential by 59mV



# Examples of charged surfaces: Oxides

- If protonation can occur at the interface (potential determining ions are  $H^+$  and  $OH^-$ )

$$K_A = \frac{[A^-][H^+]_{local}}{[AH]}$$

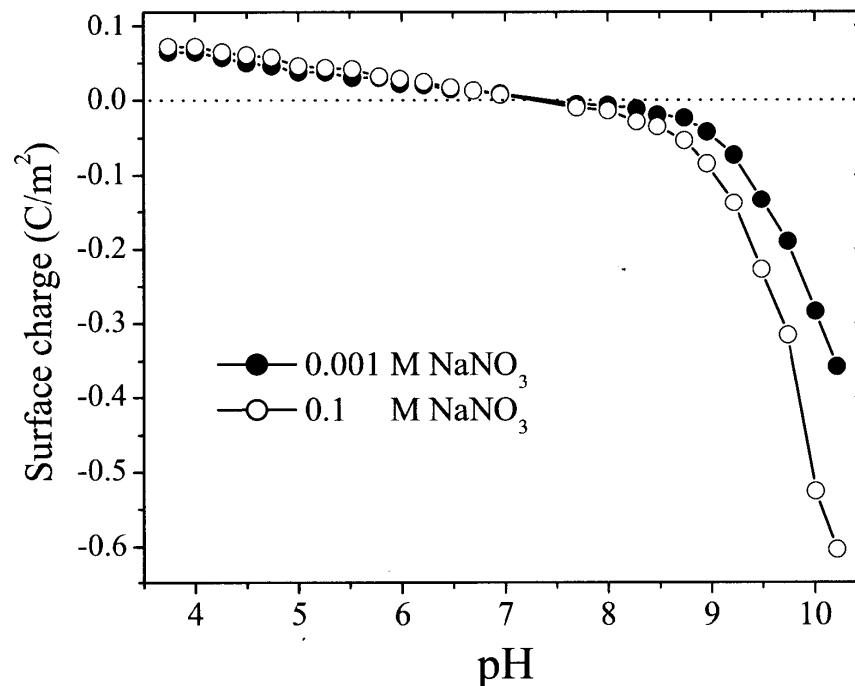
$$[H^+]_{local} = [H^+] \exp\left(-\frac{e\psi_0}{kT}\right)$$

- at 25°C, the charge at the interface will depend on pH

$$\psi_0 = 59mV \cdot \left( (pK_A - pH) + \log \frac{[A^-]}{[AH]} \right)$$

# Examples of charged surfaces: Oxides

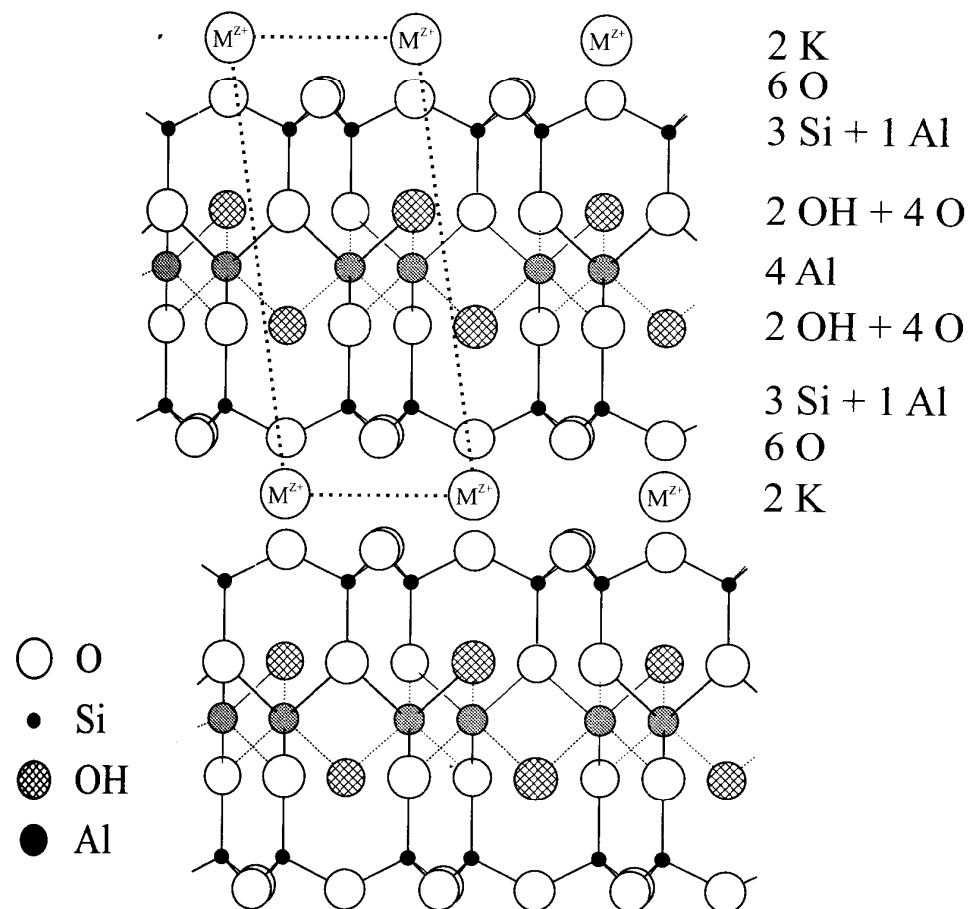
- Most of the oxides are negatively charged at neutral pH



Substance	pzc
SiO <sub>2</sub>	1.8–3.4
TiO <sub>2</sub>	2.9–6.4
Al <sub>2</sub> O <sub>3</sub>	8.1–9.7
MnO <sub>2</sub>	1.8–7.3
Fe <sub>3</sub> O <sub>4</sub>	6.0–6.9
α-Fe <sub>2</sub> O <sub>3</sub>	7.2–9.5

# Examples of charged surfaces: Mica

- Negative charge on mica surface is related to cations ( $K^+$ ,  $Si^{4+}$ ,  $Al^{3+}$  and  $Mg^{2+}$ ) leaving the surface and relatively insensitive to pH



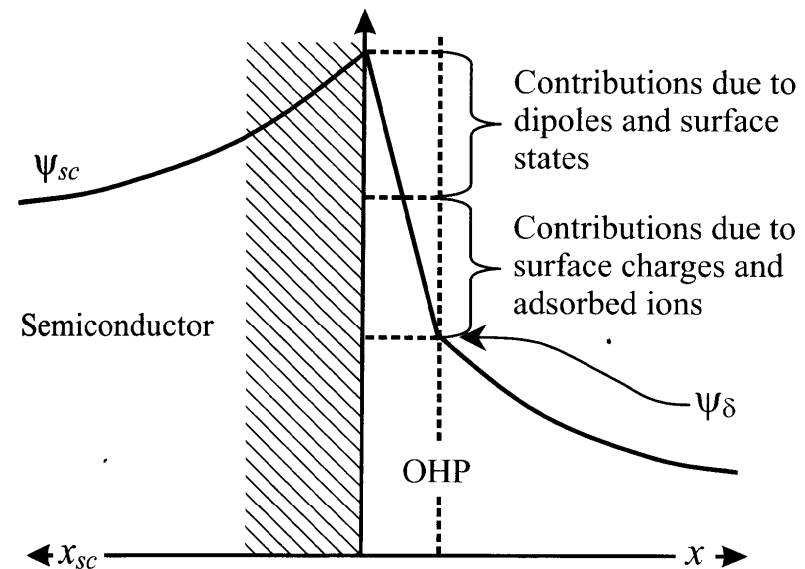
# Examples of charged surfaces: Semiconductors

- In semiconductors a smaller density of electrons leads to non-negligible distribution of electrons inside the semiconductor. Debye screening length approach can be applied to the electrons in the semiconductors

$$\lambda_D = \sqrt{\frac{\epsilon\epsilon_0 kT}{2c_e e^2}}$$

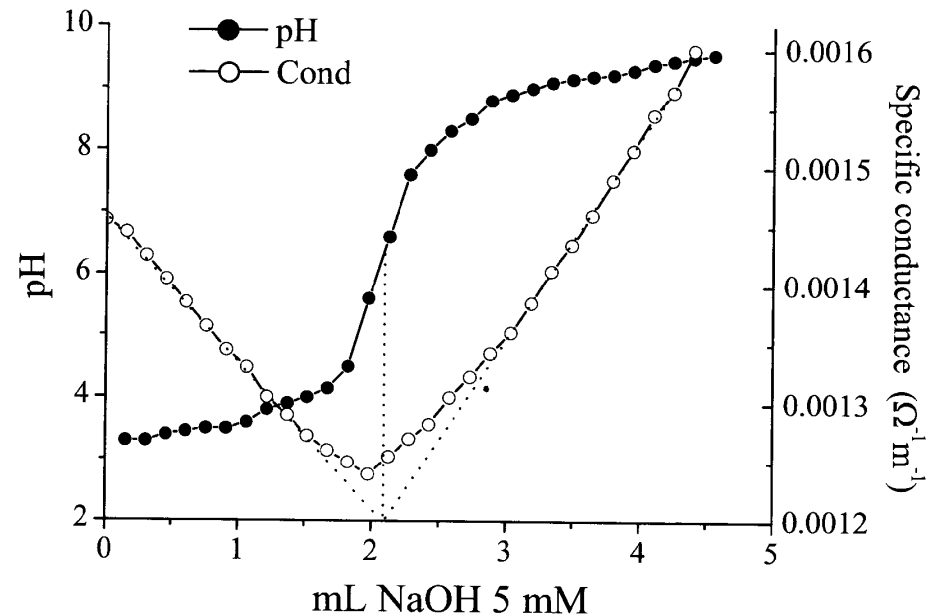
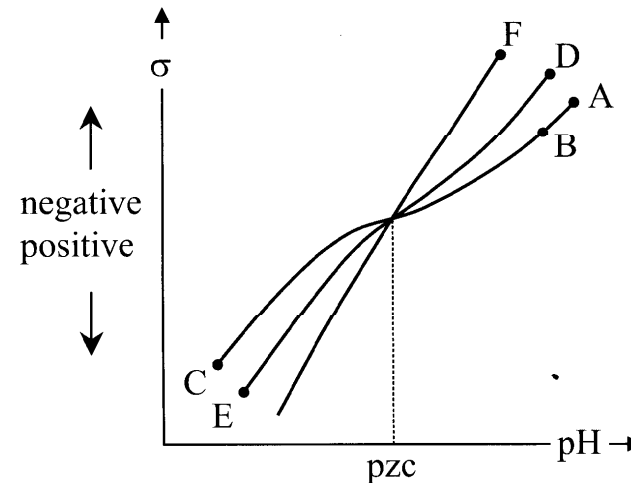
- e.g. in Ge, at 25°C

$$\lambda_D \approx 615nm \text{ (at } \epsilon = 16\text{)}$$



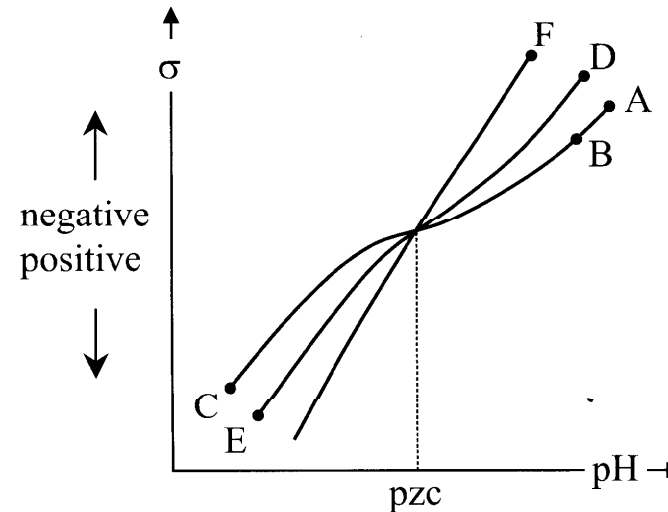
# Measuring surface charge density

- potentiometric titration
- conductimetric titration



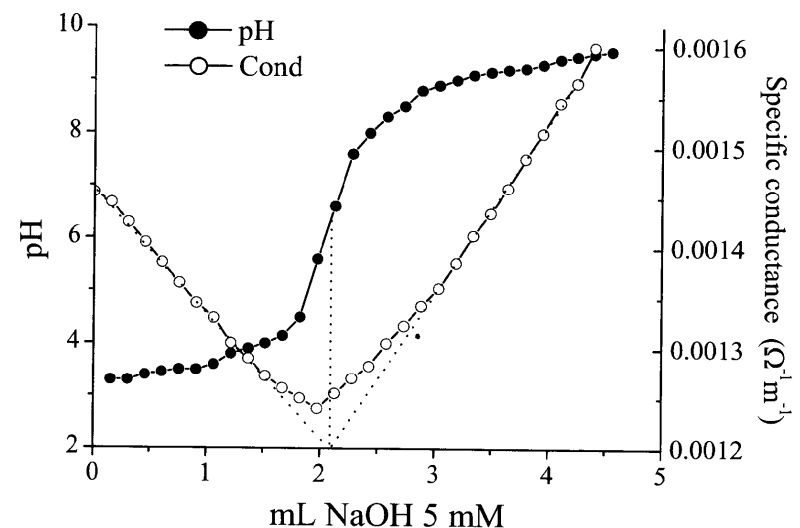
# Measuring surface charge density

- **potentiometric titration:** indifferent salt (that doesn't absorb at the surface) is used for titration of a solution of dispersed phase, e.g. KOH and  $\text{HNO}_3$ .



titration curves at increased salt concentration

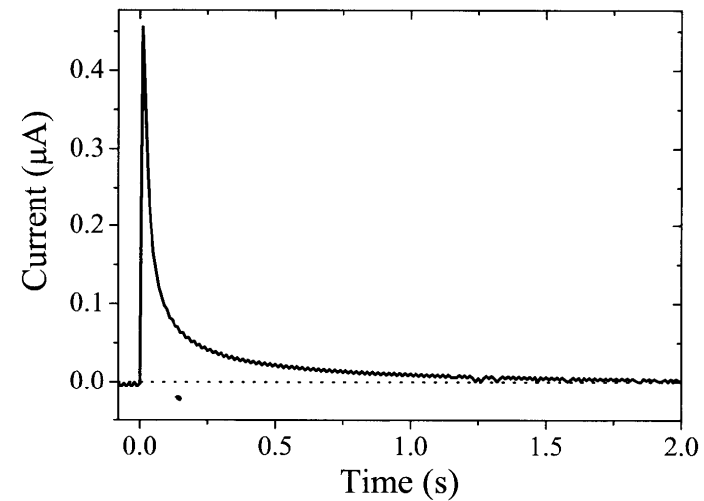
- **conductometric titration:** electrical conductivity vs. amount of potential determining ions



# Capacitance measurements

- chronoamperometric measurements

- cyclic voltammetry



- impedance spectroscopy

# Electrokinetic effects

- Navier-Stokes equations with the electric force

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho_e E_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho_e E_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho_e E_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

---

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{E} + \mu \nabla^2 \vec{V}$$

$$\nabla \cdot \vec{V} = 0$$



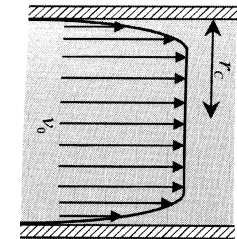
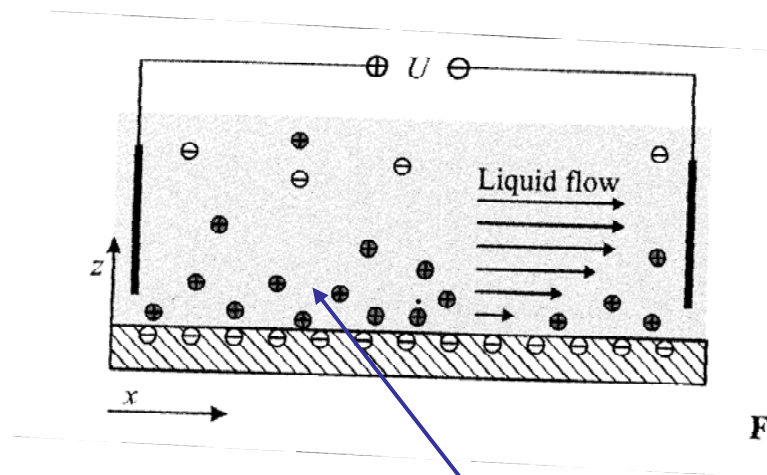
# Electrokinetic effects

- Let's consider a capillary with a voltage applied along it:

$$0 = -\frac{\partial p}{\partial x} + \rho_e E_x + \mu \frac{\partial^2 w}{\partial z^2}$$

$$0 = -\frac{\partial p}{\partial z} + \rho_e E_z + 0$$

$$\frac{\partial u}{\partial x} = 0$$



- From the Poisson equation

$$\frac{\partial^2 \psi}{\partial z^2} = -\rho_e / \epsilon \epsilon_0$$

Higher concentration of counterions next to the surface

- Combining

$$\epsilon \epsilon_0 \frac{\partial^2 \psi}{\partial z^2} E_x = \mu \frac{\partial^2 w}{\partial z^2}$$

integrating for inf to a



$$\epsilon \epsilon_0 \zeta E_x = -\mu u_0$$

# Electrokinetic effects

- Electrophoresis:
  - charged particles will move in electric field
  - the movement will be determined by the charge and the radius

$$v = v_i + v_o = \frac{QE}{6\pi\eta R} \left(1 + \lambda_D/R\right)^{-1}$$

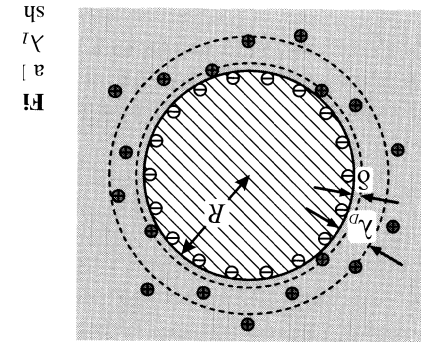
- $R \gg \lambda_D$ :

$$v = \frac{2\sigma E \lambda_D}{3\eta} = \frac{2\varepsilon\varepsilon_0 \zeta E}{3\eta}$$

$$\sigma = \varepsilon\varepsilon_0 \zeta / \lambda_D$$

- $R \ll \lambda_D$ :

$$v = \frac{\varepsilon\varepsilon_0 \zeta E}{\eta}$$



Electrophoresis is of great practical importance for charged colloidal particles. In this diagram, the electric field E is applied to the right, and the Debye length lambda\_D is indicated.

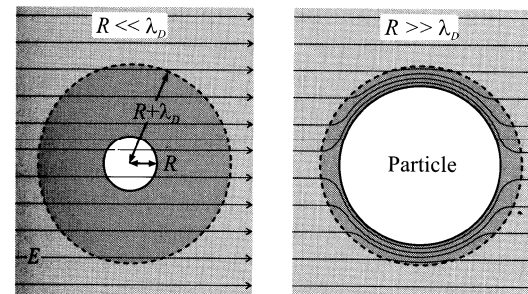


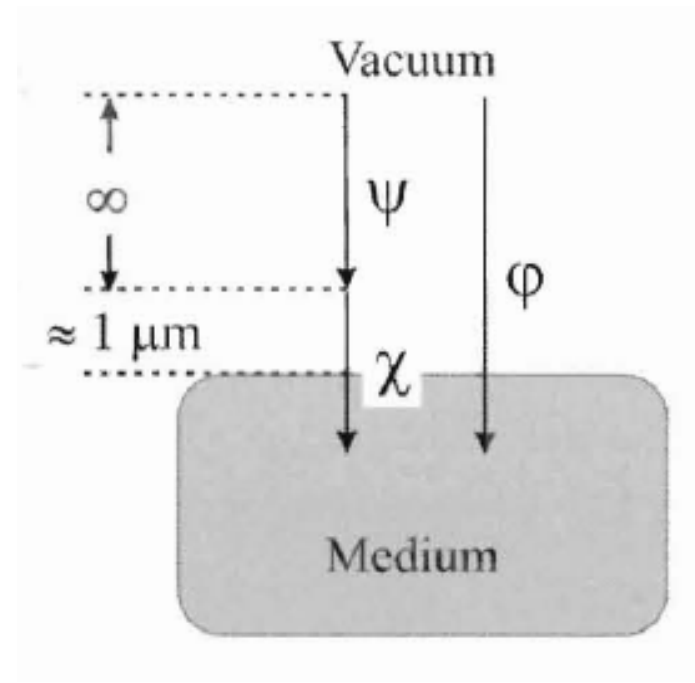
Figure 10.1: Electrophoresis of a particle in an electric field. The left diagram shows a small particle (R << lambda\_D) and the right diagram shows a large particle (R >> lambda\_D). The electric field E is applied to the right, and the Debye length lambda\_D is indicated.

# Types of Potentials

- Galvani potential  $\phi$ : work required to bring a charge from infinity into bulk
- External potential  $\psi$ : work –”  
– from infinity to the close proximity of the interface
- Surface potential (jump)  $\chi$ .

$$\phi = \psi + \chi$$

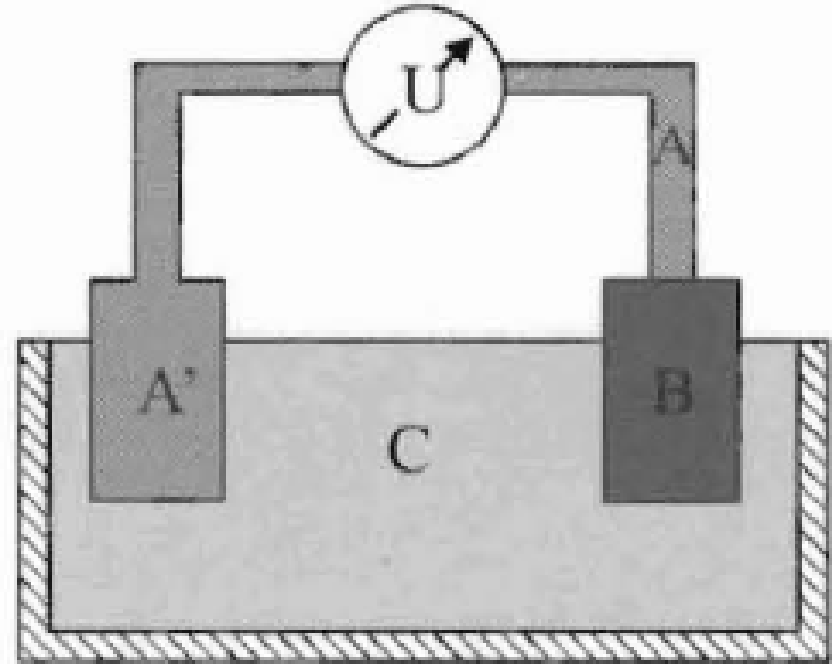
- Work function  $\Phi$ : work required to move an electrone from a Fermi level to vacuum far away from the surface



# Types of Potentials

- Potentials in an electrochemical cell

$$U = \frac{\mu_e^{A'} - \mu_e^A}{e} = \frac{\mu_e^{A'} - \mu_e^B}{e}$$



$$U = {}^{A'}\Delta^A\Phi = {}^A\Delta^B\varphi + {}^B\Delta^C\varphi + {}^C\Delta^{A'}\varphi$$

# Problems (Ch.5)

1. For a microfluidic application, a capillary of  $10\mu\text{m}$  radius and 5 cm length was fabricated in glass. The zeta potential of this glass in 0.01 M KCl aqueous solution at neutral pH is -30 mV. A potential of 5 V is applied along the capillary. How fast and in which direction does the liquid flow?
2. To observe the flow, small spherical polystyrene particles of 50 nm radius which are fluorescently labeled, are added. To keep them dispersed they have sulfate groups on their surface. This leads to a zeta potential of -20 mV. How fast and in which direction do these particles move? A good marker should move with the same speed as the liquid flow. Was it a good idea to use these particles as markers?